Name: ____________________

St George Girls High School

Trial Higher School Certificate Examination

2015

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. $\left(\frac{2a}{3b}\right)^{-5} = ?$

(A) $\frac{2a^5}{3b^5}$

(B) $\frac{3b^5}{2a^5}$

(C) $\frac{243b^5}{32a^5}$

(D) $\frac{1}{243b^5}$

2. Let α and β be the solutions of $2x^2 - 5x - 9 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(A) $-\frac{9}{2}$

(B) $-\frac{9}{5}$

(C) $-\frac{5}{9}$

(D) $\frac{5}{2}$

3. Find $\lim\limits_{x \to \infty} \frac{3\sqrt{x}}{x-2}$.

(A) $\sqrt{x}$

(B) 3

(C) $\frac{3}{x}$

(D) 0

Section I (cont'd)

4. The amplitude and period of $y = 3\cos 2x$ is:

(A) Amplitude = 2, Period = $\frac{2\pi}{3}$

(B) Amplitude = 3, Period = π

(C) Amplitude = π , Period = 3

(D) Amplitude = $\frac{2\pi}{3}$, Period = 2

5. The domain for the function $f(x) = \frac{1}{\sqrt{4-x}}$ is:

(A) $x \geq 0$

(B) $x > 4$

(C) $x < 4$

(D) all real x

6. When simplified fully $\cos^2\left(\frac{\pi}{2}-\theta\right)\cot\theta$ is:

(A) $\cos^2\theta\cot\theta$

(B) $\sin\theta\cos\theta$

(C) $\frac{\sin^3\theta}{\cos\theta}$

(D) $\sin^2\theta\cot\theta$

7. Find $\int_2^7 \frac{5}{x}\,dx$.

(A) $5(\ln 7 - \ln 2)$

(B) $\frac{1}{5}(\ln 7 - \ln 2)$

(C) $\frac{5}{49} - \frac{5}{4}$

(D) 0

Section I (cont'd)

8. The equation of the normal to the curve $x^2 = 4y$ at the point where $x = 2$ is:

   (A) $y = 1$

   (B) $x - y - 1 = 0$

   (C) $y = -1$

   (D) $y + x - 3 = 0$

9. Find the value of $\log_5 200 - 3\log_5 2$.

   (A) 1.4

   (B) 2.0

   (C) 3.2

   (D) 2.5

10. A particle is moving in a straight line. Its distance (x metres) from a fixed point O is given by $x = 2\cos 2t$, where t is the time in seconds.

    At which times is the particle at rest?

    (A) $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots\ldots$

    (B) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

    (C) $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots\ldots\ldots$

    (D) $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots\ldots\ldots$

End of Section I

Section II

90 Marks
Attempt Questions 11 – 16
All about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

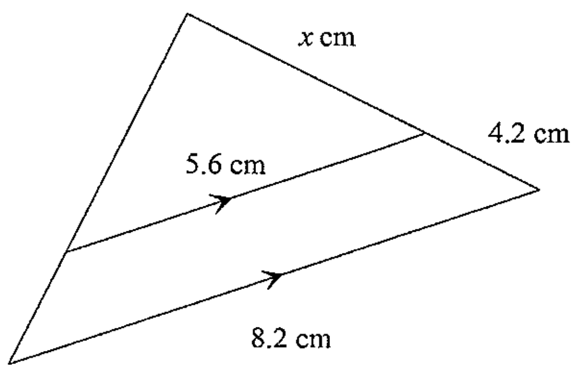
a) Solve $x^2 - 2x - 7 = 0$, expressing your answer in simplest surd form. 2

b) Find $\int \frac{3x}{x^2 + 1}\, dx$. 1

c) Simplify fully : 2

$$\frac{2}{\sqrt{7} + 3} - \frac{3\sqrt{7}}{\sqrt{7} - 3}$$

d) Find the value of x (correct to the nearest mm). 2



Question 11 continues on page 6

Question 11(continued) **Marks**

e) Find the coordinates of the vertex and focus of the parabola $x^2 - 5y + 5 = 0$. 2

f) Find the sum of the 10th to the 30th terms of the arithmetic series 5 + 9 + 13 +..... 2

g) Evaluate $\int_0^{\ln 6} e^x \, dx$. 2

h) Shade the region satisfying both the inequalities $y < \sqrt{4-(x-2)^2}$ and $y > \frac{x^2}{2}$. 2

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) Differentiate:

(i) $y = \sin^2(4x)$. 1

(ii) $y = x^3 e^{3x}$. 1

(iii) $y = \frac{e^x}{(x+3)^2}$. 2

b) Solve $\sqrt{3}\cos x = \sin x$ for $0 \le \theta \le 2\pi$. 2

c) Use Simpson's Rule with four equal subintervals to find an approximation for $\int_0^1 \tan x \, dx$. 2

d) Prove that $cosec\,\theta - \sin\theta = \cot\theta\cos\theta$. 2

e) Find the values of A, B and C if $3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$. 2

f) A curve has the equation $y = x\cos x$.

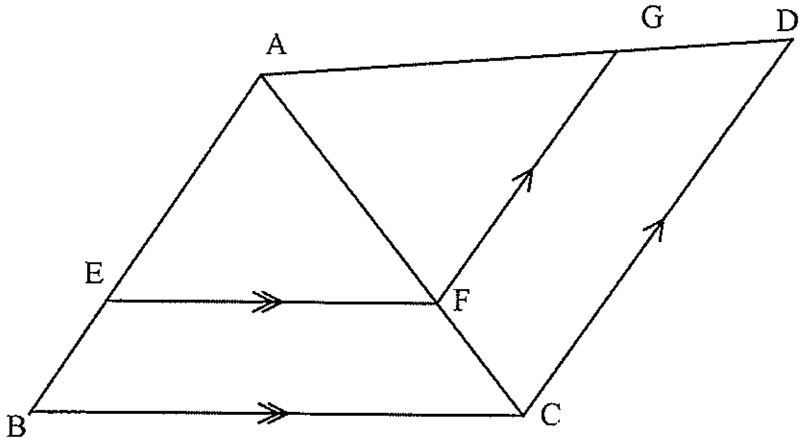
(i) Show that $P\left(\frac{\pi}{2}, 0\right)$ is the first point to the right of the origin where the curve crosses the x axis. 1

(ii) Find the equation of the tangent at point P. 2

End of Question 12

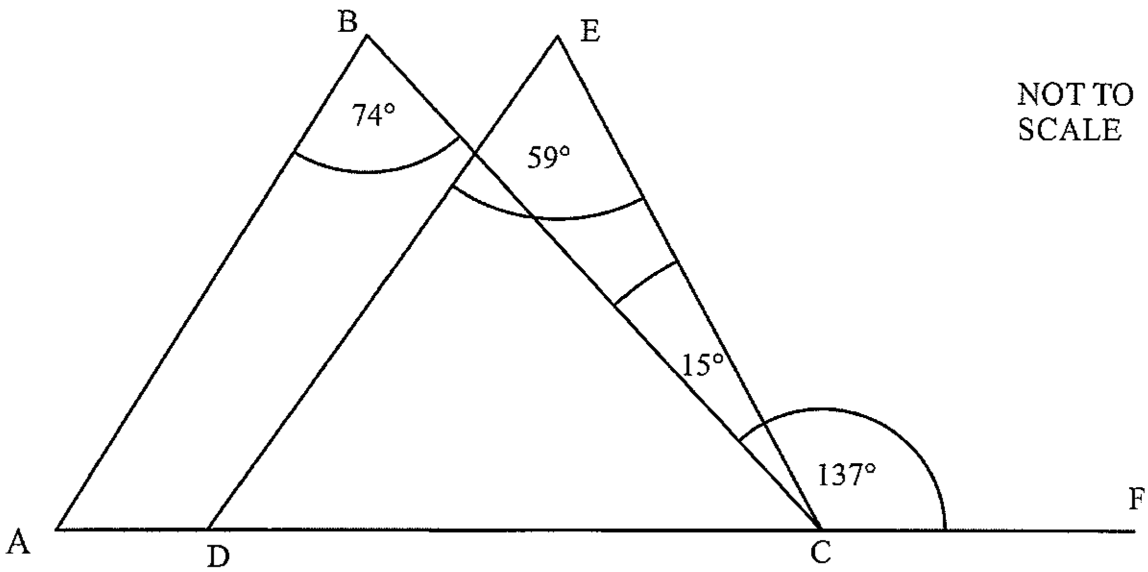
Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) In the figure below, EF || BC and CD || FG.



Prove that $\dfrac{AE}{AB} = \dfrac{AG}{AD}$. 2

b) In the diagram below AF is a straight line, $\angle B = 74°$, $\angle E = 59°$, $\angle BCF = 137°$ and $\angle BCE = 15°$.



Prove that AB||DE. 2

Question 13 continues on page 9

Question 13 (continued) **Marks**

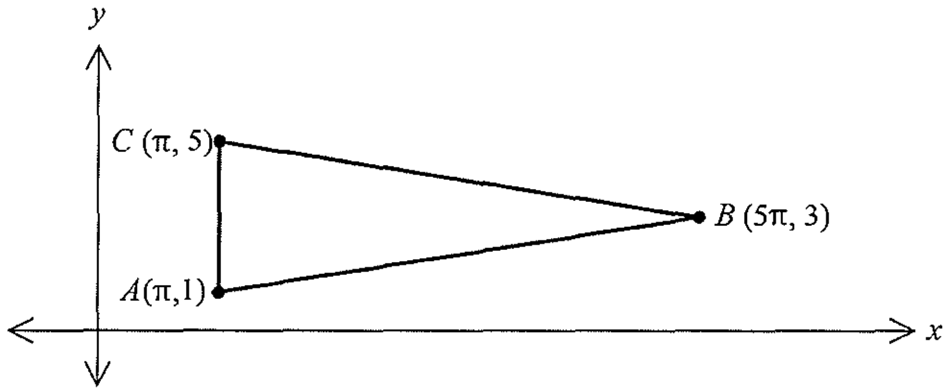
c) Jack drops a super bouncy ball from the top of a 56 m building on to a concrete surface below. Its first rebound is 42 m, and each subsequent rebound is three quarters the height of the previous one.

(i) How high will it rise on the fifth rebound? 2

(ii) How far will it travel in total? 1

d) For the domain $0 \leq x \leq 6$, a function $y = f(x)$ satisfies $f'(x) < 0$ and $f''(x) < 0$.
Sketch a possible graph of $y = f(x)$ in this domain. 2

e) The points $A\,(\pi, 1)$, $B(5\pi, 3)$ and $C(\pi, 5)$ form an isosceles triangle, with $AB = CB$.



(i) Find the midpoint of AB. 1

(ii) Show that the equation of the line which is perpendicular to AB and which passes through point C is: 2

$$y + 2\pi x - 5 - 2\pi^2 = 0$$

(iii) Calculate the distance AB. 1

(iv) Using the distances AB, BC and AC, or otherwise, find $\angle CAB$ to the nearest degree. 2

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) The relation $x^2 - 4x + y^2 = 5$ is rotated about the x-axis to form a solid. Find the exact volume of this solid of revolution. 2

b) For the curve $y = x^3(3 - x)$

(i) Find any stationary points and determine their nature. 3

(ii) Find the points of inflexion. 1

(iii) Draw a sketch of the curve showing the stationary points, inflexion points and intercepts on the axes. 2

c) Georgina borrows \$650 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 5.4% per annum reducible, calculated monthly.

(i) Show that the amount, $\$A_n$, owing after the nth repayment is given by the formula: 2

$$A_n = 650\,000(1.0045)^n - M\left(1 + 1.0045 + 1.0045^2 + \ldots\ldots + 1.0045^{n-1}\right)$$

(ii) Find the monthly repayment required to repay the loan in 30 years. 2

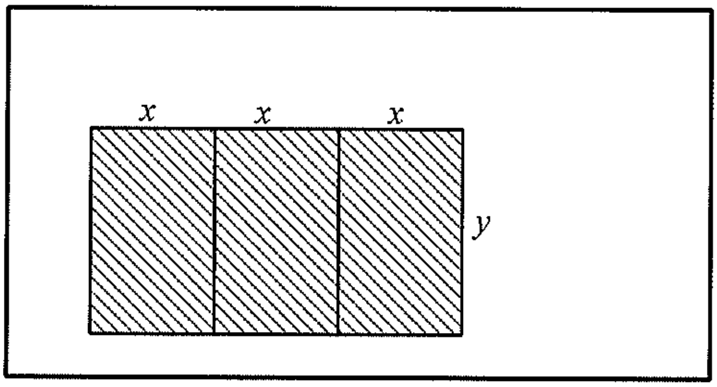
(iii) Georgina wants pay the loan off in less than 30 years. If she can afford to pay \$5 000 per month, how many months will it take her to pay off the home loan? 2

(iv) How much will Georgina save in interest if she pays \$5 000 per month? 1

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) Greg has a one hectare (ha) block of land. He is going to fence off three identical rectangular plots within his block for his three children. Each plot will measure x m by y m as shown in the diagram below. He will retain the remainder of the block for himself and his wife. Greg can only afford 300 m of fencing to go around the children's plots.



(i) Show $y = 75 - \frac{3x}{2}$. 1

(ii) Find the value of x for which the shaded area will be a maximum. 3

(iii) Find the maximum area of one of the children's blocks. 1

(iv) How much of Greg's 1 ha block is left for him and his wife? 1

Question 15 continues on page 12

Question 15 (continued) **Marks**

b) The acceleration, after t seconds, of a particle moving in a straight line is given by

$$\ddot{x} = -\frac{14}{(t+4)^3}.$$

Initially the particle is located $\frac{3}{4}$ m to the left of the origin and the initial velocity is $\frac{7}{16}$ m/s.

(i) Find the velocity v and the displacement x at any time t. 2

(ii) What is the velocity of the particle when it passes through the origin? 2

(iii) Sketch a graph of the displacement as a function of time. 2

c) Find the value of n such that: 3

$$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$$

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

a) Connor buys a new car, which begins to depreciate immediately. The value ($\$V$) of the car after t years is given by $V = A\,e^{-kt}$

Where A - is the initial value

k - constant of depreciation

t - time in years

If the car is worth \$30 000 after 5 years and \$18 000 after 10 years, find the following:

(i) The depreciation constant k. 2

(ii) The initial value of the car. 1

(iii) How many whole years will it take before the car's value falls below \$1 000? 2

b) A plane leaves an airport A and travels due north $\sqrt{3}\,x$ kilometres to a point K and then turns due west and travels a further x kilometres until it reaches a point P which is 380 kilometres from A. Due to storms the plane is then diverted to a new airport B which is 200 kilometres on a bearing of 280° from A.

(i) Draw a diagram and label it to show the above information. 1

(ii) Find the exact distance AK. 1

(iii) Show that the plane needs to travel 294 kilometres from P to the new airport B. 2

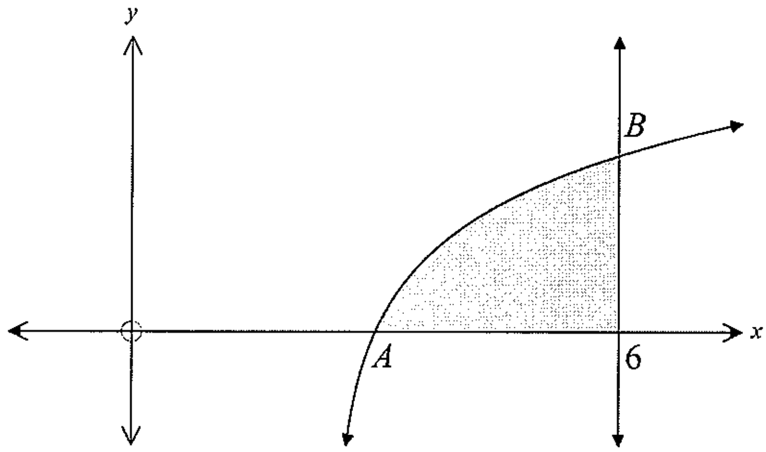
(iv) Hence or otherwise find the bearing (to the nearest degree) on which the plane flies from P to B. 1

Question 16 continues on page 14

Question 16 (continued) **Marks**

c) The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersects the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are $(3, 0)$ and $(6, \ln 7)$ respectively. 2

(ii) Show that if $y = \ln(2x - 5)$, then $x = \dfrac{e^y + 5}{2}$. 1

(iii) Hence, show the exact area of the shaded region is $\frac{1}{2}(7\ln 7 - 6)$ square units. 2

End of Examination

TRIAL - 2015 Mathematics Paper

Section I - Solutions & Answers

1. $\left(\frac{2a}{3b}\right)^{-5} = \left(\frac{3b}{2a}\right)^{5}$

$= \frac{243b^5}{32a^5}$ — C

2. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-b/a}{c/a}$

$a=2$
$b=-5$
$c=-9$

$= \frac{-b}{c}$

$= \frac{--5}{-9}$

$= -\frac{5}{9}$ — C

3. $\lim_{x\to\infty} \frac{3\sqrt{x}}{x-2}$

$= \lim_{x\to\infty} \frac{\frac{3\sqrt{x}}{x}}{\frac{x}{x} - \frac{2}{x}}$

$= \lim_{x\to\infty} \frac{\frac{3\sqrt{x}}{x}}{1-\frac{2}{x}}$

$= \frac{0}{1-0}$

$= \frac{0}{1}$

$= 0$ — D

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\frac{3\sqrt{x}}{x} = 3\sqrt{x} \times \frac{1}{x}$

4. $y = 3\cos 2x$

$a = 3$
$n = 2$

period $= \frac{2\pi}{n}$

$= \frac{2\pi}{2}$

$= \pi$ — B

5. $f(x) = \frac{1}{\sqrt{4-x}}$

D: $4 - x > 0$
$4 > x$
$\therefore x < 4$ — C

6. $\cos^2\left(\frac{\pi}{2} - \theta\right)\cot\theta$

$\cos(90^\circ - \theta)$
$= \sin\theta$

$= \sin^{\cancel{2}}\theta \times \frac{\cos\theta}{\cancel{\sin\theta}}$

$= \sin\theta\cos\theta$ — B

7. $\int_2^7 \frac{5}{x} \, dx$

$= 5\left[\ln x\right]_2^7$

$= 5(\ln 7 - \ln 2)$ — A

8.

$$x^2 = 4y$$
$$y = \frac{x^2}{4}$$
$$\frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2}$$

when $x=2$, $\frac{dy}{dx} = \frac{2}{2} = 1$

$$m_T = 1$$
$$\therefore m_N = -1$$

$x = 2$, $y = \frac{2^2}{4} = \frac{4}{4} = 1$

$\therefore (2, 1)$

Eq$^{\underline{n}}$ of normal:

$$y - 1 = -1(x-2)$$
$$y - 1 = -x + 2$$
$$x + y - 3 = 0 \quad \text{—— D}$$

9.

$$\log_5 200 - 3\log_5 2$$
$$= \log_5 200 - \log_5 2^3$$
$$= \log_5 \left(\frac{200}{8}\right)$$
$$= \log_5 25 \qquad \text{as } 5^2 = 25$$
$$= 2 \quad \text{—— B}$$

10.

$$x = 2\cos 2t$$
$$V = \frac{dx}{dt} = 2 \times -\sin 2t \times 2$$
$$= -4\sin 2t$$

at rest when $V = 0$,

ie $-4\sin 2t = 0$

$$\sin 2t = 0$$
$$2t = 0°, 180°, 360°, 540°, \ldots$$
$$t = 0°, 90°, 180°, 270°, \ldots$$
$$= 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots \quad \text{—— A}$$

Summary:

1.	C	6.	B
2.	C	7.	A
3.	D	8.	D
4.	B	9.	B
5.	C	10.	A

Section II

Question 11. (15 marks)

(a) $x^2 - 2x - 7 = 0$

$$x^2 - 2x + \left(-\tfrac{2}{2}\right)^2 = 7 + \left(-\tfrac{2}{2}\right)^2$$
$$(x-1)^2 = 7 + 1$$
$$(x-1)^2 = 8 \qquad \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$$
$$x - 1 = \pm\sqrt{8}$$
$$x = 1 \pm \sqrt{8} \quad — ①$$
$$\therefore x = 1 \pm 2\sqrt{2} \quad — ①$$

OR $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad a = 1,\ b = -2,\ c = -7$

$$= \frac{--2 \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4 + 28}}{2}$$
$$= \frac{2 \pm \sqrt{32}}{2} \quad — ① \qquad \sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$$
$$= \frac{2 \pm 4\sqrt{2}}{2}$$
$$= \frac{\cancel{2}(1 \pm 2\sqrt{2})}{\cancel{2}}$$
$$\therefore x = 1 \pm 2\sqrt{2} \quad — ①$$

Markers comments:

(a) generally well done by most students

Common problems were:
- not knowing how to complete the square
- not knowing correct quadratic formula
- not simplifying $\sqrt{8}$ to $2\sqrt{2}$ or $\sqrt{32}$ to $4\sqrt{2}$
- students cancelling incorrectly

  eg $\dfrac{\cancel{2} \pm 4\sqrt{2}}{\cancel{2}} = 1 \pm 4\sqrt{2}$ ✗ (need to factorise first)

Marking criteria
- provides a correct solution – 2 marks
- $x = 1 \pm \sqrt{8}$ – 1 mark
- $x = \dfrac{2 \pm \sqrt{32}}{2}$ – 1 mark

(b) $\int \frac{3x}{x^2+1} \cdot dx$

$= \frac{3}{2} \int \frac{2x}{x^2+1} \cdot dx$

$= \frac{3}{2} \ln(x^2+1) + C$ — ① correct answer only.

(c) $\frac{2}{\sqrt{7}+3} - \frac{3\sqrt{7}}{\sqrt{7}-3}$

$= \frac{2(\sqrt{7}-3) - 3\sqrt{7}(\sqrt{7}+3)}{(\sqrt{7}+3)(\sqrt{7}-3)}$

$= \frac{2\sqrt{7} - 6 - 21 - 9\sqrt{7}}{7-9}$

①

$= \frac{-7\sqrt{7} - 27}{-2}$

$= \frac{7\sqrt{7} + 27}{2}$ — ① fully simplified

(b) the majority of students scored full marks for this part

Marking criteria : 1 mark for correct answer

Common problems were:

- students writing $\frac{2}{3}$ out the front of the log instead of $\frac{3}{2}$.
- not recognising it was a log question

(c) mostly well done by students

Common problems were:

- girls not expanding 2nd bracket correctly leading to $-21 + 9\sqrt{7}$
- some girls stopped at $\frac{-7\sqrt{7}-27}{-2}$ — 1 mark only

Marking criteria:

- provides fully simplified solution — 2 marks
- $\frac{-7\sqrt{7}-27}{-2}$ — 1 mark
- $\frac{2\sqrt{7}-6-21-9\sqrt{7}}{7-9}$ — 1 mark

(d) Triangles are similar (equiangular)

$\therefore \dfrac{x}{5.6} = \dfrac{x+4.2}{8.2}$ (corresponding sides of similar triangles are in the same ratio)

① for correct ratio

$8.2x = 5.6(x+4.2)$

$8.2x = 5.6x + 23.52$

$2.6x = 23.52$

$x = \dfrac{23.52}{2.6}$

$= 9.046153846$

$= 9.0$ cm

$= 90$ mm

— ①

(d) this part generally well done

Common problems were:

- students not writing out complete calculator answer, $x = 9.046153846$ this made marking the final answer difficult, I had nothing to check.
- students still cannot round off to the nearest mm, ie 1 decimal place 9.0 cm or 90 mm (multiplying by 10)

Marking criteria:

- provides a correct solution – 2 marks
- correct ratio – 1 mark
- $x = 9.046153846$ $= 9.0$cm OR $= 90$ mm – 1 mark

- students did not lose marks if they didn't state the triangles were similar first, thumbs up to the students that did!
- no marks deducted if wrong rounding (students → learn how to round off please!
  - show all your calculator answer, none of this 9.04 ...)

(e) $x^2 - 5y + 5 = 0$

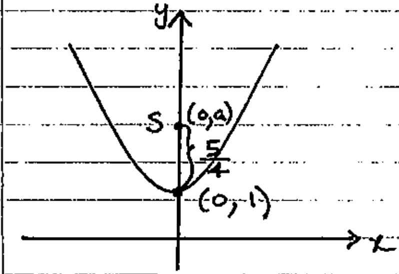
$x^2 = 5y - 5$

$= 5(y-1)$

$\therefore$ Vertex is $(0,1)$ — ① correct answer

Focal length : $4a = 5$

$a = \frac{5}{4}$



Focus $= \left(0, 1+\frac{5}{4}\right)$

$= \left(0, 2\frac{1}{4}\right)$ — ①

$= \left(0, \frac{9}{4}\right)$ correct answer

(e) this part not done well by some students

Common problems were:

- students not keeping the question simple and making x^2 the subject to get equation in the form $(x-h)^2 = 4a(y-k)^2$
  ie $x^2 = 5(y-1)$
  $\therefore$ Vertex is $(0,1)$ straightaway
- students complicated question by completing the square and didn't do this correctly and thus yielding wrong answers.
- students not recognising parabola is concave up!
- focus needed to have the same x coordinate as vertex and then $1+\frac{5}{4}$ for the y-coordinate.
- some students didn't find focal length to help get the coordinates of the focus.

Marking criteria :

- 1 mark for correct vertex
- 1 mark for correct focus
- 1 mark (CFPA) for correct focus from wrong vertex, using correct focal length, had to have the same x-coordinate as vertex.

(f) $5 + 9 + 13 + \dots$

$a = 5$, $d = 4$

$T_n = a + (n-1)d$

$T_9 = a + 8d$
$= 5 + 8(4)$
$= 37$

$T_{10} = 37 + 4$
$= 41$ — (1)

$T_{30} = a + 29d$
$= 5 + 29(4)$
$= 121$

$\underbrace{5 + 9 + 13 + \dots + 37}\ + \underbrace{41 + 45 + \dots + 121}$

$\therefore$ Sum of the 10^{th} to the 30^{th} terms

$= \text{Sum to } 30 - \text{Sum to } 9$ — (1)

$= \frac{30}{2}(5 + 121) - \frac{9}{2}(5 + 37)$ $\qquad S_n = \frac{n}{2}(a + l)$

$= 1890 - 189$

$= 1701$ — (1)

OR $\overset{10^{th}}{}\qquad\qquad\overset{30^{th}}{}$

$\underbrace{41 + 45 + \dots + 121}_{n = 21}$

$n = 30 - 10 + 1$
$= 21$

using

$S_n = \frac{n}{2}(a + l)$

$a = 41$
$l = 121$

$S_{21} = \frac{21}{2}(41 + 121)$

$= \frac{21}{2} \times 162$

$= 1701$ — (2)

(f) generally well done by most students

Common problems were: • wrong formulas!
• prime example were students don't read the question. Just finding T_{10} & T_{30} was not the question. Sum of the 10^{th} to 30^{th} terms ie $41 + 45 + \dots + 121$ which has 21 terms not 20. $(n = 30 - 10 + 1 = 21)$

Marking Criteria

- provides a correct solution – 2 marks
- $T_{10} = 41$ & $T_{30} = 121$ – 1 mark
- $S_{30} - S_{10}$ – 1 mark (should have $S_{30} - S_9$)
- S_{20} – 1 mark
- S_{30} – 1 mark

(g) $\int_0^{\ln 6} e^x \cdot dx$

$= \left[e^x\right]_0^{\ln 6}$

$= e^{\ln 6} - e^0$

$= 6 - 1$

$= 5$

(h) $y = \frac{x^2}{2}$ concave up parabola

$y = \sqrt{4-(x-2)^2} \Rightarrow$ semi circle

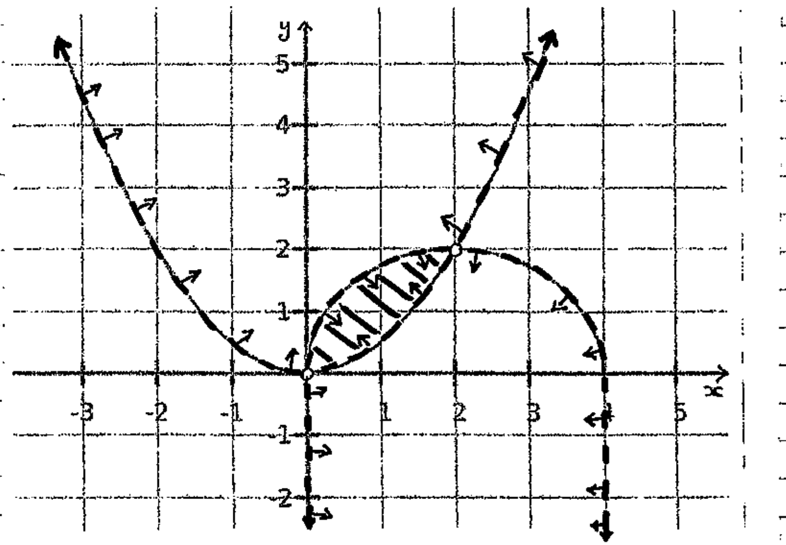
$y^2 = 4-(x-2)^2$

$(x-2)^2 + y^2 = 4$

centre $(2, 0)$

radius $= 2$

Method ① using $y < \sqrt{4-(x-2)^2}$



test $(2,-3)$:

$-3 < \sqrt{4-(2-2)^2}$

$-3 < \sqrt{4}$

$-3 < 2$ ✓

test $(2,4)$: $y > \frac{x^2}{2}$

$4 > \frac{4}{2}$

$4 > 2$ ✓

$y < \sqrt{4-(x-2)^2}$

$4 < \sqrt{4-(2-2)^2}$

$4 < \sqrt{4}$

$4 < 2$ ✗

(g) majority of students scored full marks for this part

Common problems were:

- students still don't know $e^{\ln x} = x$ & $e^0 = 1$

Marking criteria:

- provides a correct solution – 2 marks
- an answer of 6 with working – 1 mark

(h) • some students did not attempt this question

• those that did, a reasonable job was done.

Common problems were:

- students did not recognise $y = \frac{x^2}{2}$ as a parabola ✓. and $y = \sqrt{4-(x-2)^2}$ as a semi-circle with Centre $(2, 0)$ and radius $= 2$ OR $y^2 = 4-(x-2)^2$ as a complete circle
- students did not make both curves dotted lines and the whole curves.
- no testing of points
- 2 methods possible and both yield same answer, see solutions!

Marking criteria:

- provides correct solution – 2 marks
- correct drawing of both curves with dotted lines – 1 mark
- correct shading of intersection – 1 mark
- no marks deducted if students didn't have open circles at $(0,0)$ and $(2,2)$.
- students lost 1 mark if they only drew half of the parabola.

Mathematics trial 2015

18 2015

Question 12

a) Differentiate

Comments

(i) $y = \sin^2 4x$

$y = [\sin(4x)]^2$

$\frac{dy}{dx} = 2 \cdot \sin(4x) \times \cos(4x) \times 4$

$= 8\sin(4x)\cos(4x)$

1 mark. Some extension candidates simplified to $\frac{dy}{dx} = 4\sin 8x$

(ii) $y = x^3 e^{3x}$

$u = x^3$, $v = e^{3x}$, $\frac{du}{dx} = 3x^2$, $\frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= 3x^2 e^{3x} + 3x^3 e^{3x}$

$= 3x^2 e^{3x}(1+x)$

1 mark

(iii) $y = \frac{e^x}{(x+3)^2}$

$u = e^x$, $v = (x+3)^2$, $\frac{du}{dx} = e^x$, $\frac{dv}{dx} = 2(x+3) = 2x+6$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{e^x(x+3)^2 - 2e^x(x+3)}{(x+3)^4}$

$= \frac{e^x(x+3)[x+3-2]}{(x+3)^4}$

$= \frac{e^x(x+1)}{(x+3)^3}$

2 marks

Many candidates incorrectly stated the formula

- u was confused with v
- $+$ instead of $-$
- many students didn't fully simplify answer and lost some marks.

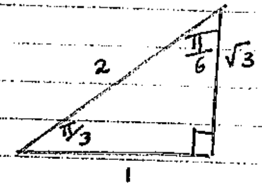
(b) Show $\sqrt{3}\cos x = \sin x$ for $0 \le x \le 2\pi$

Comments

2 marks

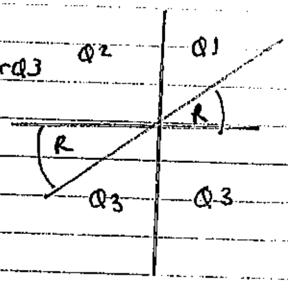
$\sqrt{3} = \frac{\sin x}{\cos x}$

$\therefore \tan x = \sqrt{3}$



$R = \frac{\pi}{3}$

x lies in Q1 or Q3



$\therefore x = \frac{\pi}{3}$ or $\pi + \frac{\pi}{3}$

$= \frac{\pi^c}{3}$ or $\frac{4\pi^c}{3}$

Students that used squaring in their solutions, more often than not, forget to check to see if all these were actual solutions.

(c) Simpsons Rule

$\int_0^1 \tan x \, dx$

- 2 equal subintervals
- 5 function values
- 2 applications

x	0	0.25	0.5	0.75	1
$f(x)$	tan 0	tan 0.25	tan 0.5	tan 0.75	tan 1

$A \div \frac{h}{3}[d_f + 4d_m + d_l]$

or

$\frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$= \frac{0.25}{3}[\tan 0 + 4\tan(0.25) + \tan 0.5]$

$+ \frac{0.25}{3}[\tan 0.5 + 4\tan 0.75 + \tan 1]$

$= 0.61649051$

$= 0.6$ (1 d.p.)

3 marks

- Many students received 0 for this question as they did not use an odd number of function values.
- a large number of students lost a mark for using degrees instead of radians
- Various versions of the formula were used. some students confused the different methods and were unsuccessful

Comments

d) Prove

$\operatorname{cosec}\theta - \sin\theta = \cot\theta\cos\theta$

LHS $\operatorname{cosec}\theta - \sin\theta$

$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1}$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta}\cdot\cos\theta$

$= \cot\theta\cdot\cos\theta$

$=$ RHS

$\therefore \operatorname{cosec}\theta - \sin\theta = \cot\theta\cos\theta$

2 Marks
- generally well done
- weaker students did not separate the LHS and RHS clearly & did not receive full marks.
- all steps must be shown.

(e) $3x^2+x+1 \equiv A(x-1)(x+2)+B(x+1)+C$

RHS $= A(x^2+x-2)+Bx+B+C$

$= Ax^2+Ax-2A+Bx+B+C$

$= Ax^2+(A+B)x+(B+C-2A)$

$=$ LHS

$\therefore$ equationing coefficients...

$A=3$ —— ①

$A+B=1$ —— ②

$B+C-2A=1$ —— ③

sub. ① in ② $3+B=1$

$\therefore B=-2$ —— ④

sub ① & ④ in ③

$-2+C-2(3)=1$

$C=1+6+2$

$=9$

$\therefore A=3,\ B=-2,\ C=9.$

2 marks
- generally well done
- some marks lost due to errors in calculators
- equating coefficients was the most successful method although some students successfully used substitution.

13.

(a) $\frac{AE}{AB} = \frac{AF}{AC}$ (ratio of intercepts) (or If a line is parallel to one side of a triangle then it divides the other two sides proportionally)

$\frac{AF}{AC} = \frac{AG}{AD}$ (ratio of intercepts)

$\therefore \frac{AE}{AB} = \frac{AG}{AD}$ (both equal to $\frac{AF}{AC}$) —①

(b) $B\hat{A}D = 137^\circ - 74^\circ$ (exterior angle equals the sum of the two interior opposite angles)

$= 63^\circ$

Similarly,

$E\hat{D}C = (137^\circ - 15^\circ) - 59^\circ$

$= 63^\circ$

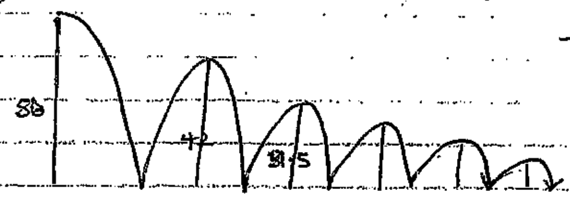
$\therefore B\hat{A}D = E\hat{D}C$ (both equal to 63°)

$\therefore AB \parallel DE$ (a pair of corresponding angles equal).

① for steps.

① for appropriate reason at all steps.

(c)



* loss of ① mark if solved with $a=56$ & $n=5$

(i) $a = 56, \; r = \frac{3}{4}$

$T_n = ar^{n-1}$

$T_6 = ar^5$

$= 56 \times \left(\frac{3}{4}\right)^5$

$= 13.2890625$

$= 13.29$ m (to 2 dec. pls.)

OR $a = 42, \; r = \frac{3}{4}$

$T_5 = ar^4$

$= 42 \times \frac{3}{4}^4$

$= 13.29$ m.

(ii) $|r| < 1$ for limiting sum

$\left|\frac{3}{4}\right| < 1$ ✓

$S_\infty = \frac{a}{1-r}$

Total travelled $= 56 + 84 + 63 + \ldots$

$= 56 + \frac{84}{1-\frac{3}{4}}$

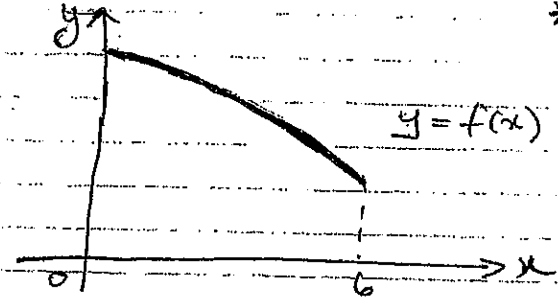
$= 56 + 336$ ①

$= 392$ m.

(d) $0 \le x \le 6$

$f'(x) < 0$ (−ve gradient of tangent, decreasing)

$f''(x) < 0$ (concave down)



* Make sure the graph is not horizontal at $x=0$ & vertical at $x=6$

(e)

(i) Midpoint of $AB = \left(\frac{\pi + 5\pi}{2}, \frac{1+3}{2}\right)$

$= (3\pi, 2)$

(ii) $m_{AB} = \frac{3-1}{5\pi - \pi} = \frac{2}{4\pi} = \frac{1}{2\pi}$

* loss of 1 mark if gradient incorrect.

$m_1 = \frac{1}{2\pi} \quad \therefore m_2 = -2\pi$

$C(\pi, 5)$

Eqn: $y - 5 = -2\pi(x - \pi)$

$y - 5 = -2\pi x + 2\pi^2$

$2\pi x + y - 5 - 2\pi^2 = 0$ as required.

(iii) $AB = \sqrt{(5\pi - \pi)^2 + (3-1)^2}$

$= \sqrt{(4\pi)^2 + 2^2}$

$= \sqrt{16\pi^2 + 4}$ ①

* if students made $\sqrt{16\pi^2+4} = 4\pi + 2$

(iv) $AB = BC = \sqrt{16\pi^2+4}$

$AC = 5-1 = 4$

$$\cos A = \frac{16\pi^2+4+4^2-(16\pi^2+4)}{2\times\sqrt{16\pi^2+4}\times 4}$$ (1)

$$= \frac{\cancel{16\pi^2}+20-\cancel{16\pi^2}-4}{8\sqrt{16\pi^2+4}}$$

$$= \frac{16}{8\sqrt{16\pi^2+4}}$$

$$\cos A = \frac{2}{\sqrt{16\pi^2+4}}$$

$\hat{A} = 80^\circ\ 57'\ 24\cdot 98''$

$\therefore\ C\hat{A}B = 81^\circ$ (to the nearest degree) (1)

OR $m = \tan\theta$

$\tan\theta = \frac{1}{2\pi}$

$\theta = 9^\circ\ 2'\ 35\cdot 02''$

$\theta = 9^\circ$

$\therefore\ C\hat{A}B = 90^\circ - 9^\circ$

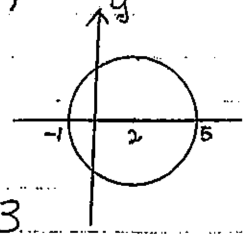
$= 81^\circ$.

(14.) $x^2 - 4x + y^2 = 5$

(a). $x^2 - 4x + \left(\frac{-4}{2}\right)^2 + y^2 = 5 + \left(\frac{-4}{2}\right)^2$

$(x-2)^2 + y^2 = 5+4$

$= 9$



$\therefore$ Circle centre $(2,0)$ $r=3$

Solid formed will be a sphere

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\times 3^3$ 2

$= 36\pi$ cubic units

OR $y^2 = -x^2+4x+5$

$$V = \pi\int_{-1}^{5} y^2\,dx$$

$$= \pi\int_{-1}^{5} -x^2+4x+5\ dx$$

$$= \pi\left[\frac{-x^3}{3}+\frac{4x^2}{2}+5x\right]_{-1}^{5}$$

$$= \pi\left[\frac{-5^3}{3}+2(5)^2+5(5) - \left(-\frac{(-1)^3}{3}+2(-1)^2+5(-1)\right)\right]$$

$$= \pi\left[\frac{-125}{3}+50+25 - \left(\frac{1}{3}+2-5\right)\right]$$

$= 36\pi$ cubic units.

most students used this approach. mistakes were made in finding the boundaries

(b) $y = x^3(3-x)$

(i) $y = 3x^3 - x^4$

$y' = 9x^2 - 4x^3$

$y'' = 18x - 12x^2$

Stationary pts occur when $y' = 0$

ie $9x^2 - 4x^3 = 0$

$x^2(9 - 4x) = 0$

$x = 0$ $\quad$ $4x = 9$

$x = \frac{9}{4}$

$x = 0$
$y = 0$

$x = \frac{9}{4}$

$y = \left(\frac{9}{4}\right)^3\left(3 - \frac{9}{4}\right)$

$= \frac{729}{64} \times \frac{3}{4}$

$= 8\frac{139}{256}$ $\quad (8.54296875)$

When $x = 0$, $y'' = 18(0) - 12(0)^2$

$= 0$

$\therefore (0,0)$ is a possible point of horizontal inflexion.

3

x	-1	0	1
y''	-30	0	6

since the concavity changes

$(0,0)$ is a horizontal point of inflexion

students rarely found this horizontal inflexion so lost 1 mark which was not earned though

When $x = \frac{9}{4}$ $\quad y'' = 18\left(\frac{9}{4}\right) - 12\left(\frac{9}{4}\right)^2$

$= -20\frac{1}{4} < 0$ $\quad \cap$

$\therefore \left(\frac{9}{4}, 8.543\right)$ is a maximum turning point.

$\left(2\frac{1}{4}, 8.543\right)$

(ii) Points of inflexion occur when $y'' = 0$

ie $18x - 12x^2 = 0$

$6x(3 - 2x) = 0$

$x = 0$ $\quad$ $x = 1\frac{1}{2}$

$y = 0$ $\quad$ $y = \left(\frac{3}{2}\right)^3\left(3 - \frac{3}{2}\right)$

$(0,0)$ is a horizontal pt of inflexion from (i)

$= \frac{27}{8} \times \frac{3}{2}$

$= 5\frac{1}{16}$

1

x	1	$1\frac{1}{2}$	2
y''	6	0	-12

since the concavity changes

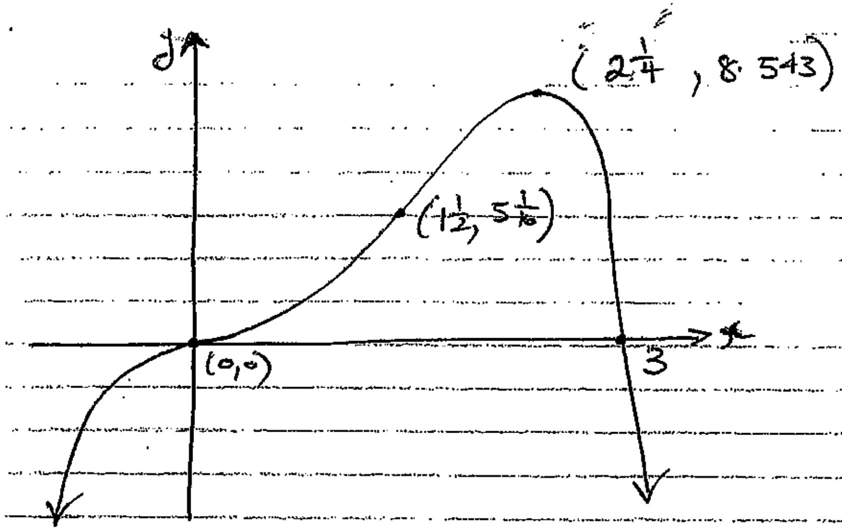
$\left(1\frac{1}{2}, 5\frac{1}{16}\right)$ is a pt of inflexion.

(iii)

Intercepts: $y = 0$, $\quad x^3(3-x) = 0$

$x = 0$, $\quad x = 3$

$y = 0$ $\quad y = 0$



2

(c) $n = 30 \times 12 = 360$

$r = 5.4\%_{pa} = 0.45\%\ pm$
$= 0.0045$

(i) Let the monthly repayment be M and the money owing after the n^{th} payment be A_n

$A = P(1+r)^n - M$

$A_1 = 650\,000\,(1.0045)^1 - M$

2

$A_2 = A_1\,(1.0045)^1 - M$

$= \boxed{650\,000(1.0045) - M}\,(1.0045) - M$

$= 650\,000(1.0045)^2 - M(1.0045) - M$

$= 650\,000(1.0045)^2 - M(1 + 1.0045)$

$A_3 = A_2\,(1.0045) - M$

$= \boxed{650\,000\,(1.0045)^2 - M(1 + 1.0045)}\,1.0045 - M$

$= 650\,000\,(1.0045)^3 - M(1.0045)(1 + 1.0045) - M$

$= 650\,000\,(1.0045)^3 - M\left[1.0045 + 1.0045^2\right] - M$

$A_3 = 650\,000\,(1.0045)^3 - M\left[1 + 1.0045 + 1.0045^2\right]$

$\vdots$

$A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + 1.0045^2 + \dots + 1.0045^{n-1}\right]$

(ii) The loan is repaid in 360 months

ie $A_{360} = 0$

mistakes were made in this step

$\therefore 650\,000(1.0045)^{360} - M\underbrace{\left[1 + 1.0045 + \dots + 1.0045^{359}\right]}_{a\ GP}$

$a = 1 \quad r = 1.0045$

$n = 360$

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$M\left[\dfrac{1.0045^{360} - 1}{0.0045}\right] = 650\,000(1.0045)^{360}$

$M = \dfrac{650\,000(1.0045)^{360}}{\left[\dfrac{1.0045^{360} - 1}{0.0045}\right]}$

$= 650\,000(1.0045)^{360} \times \dfrac{0.0045}{1.0045^{360} - 1}$

$= 3649.950147$

$\therefore M = \$3649.95$ #

2

(iii) $A_n = 0$ when paid off

When $M = 5000$

$650\,000(1.0045)^n - 5000\left[\dfrac{1.0045^n - 1}{0.0045}\right] = 0$

$650\,000(1.0045)^n = 5000\left[\dfrac{1.0045^n - 1}{0.0045}\right]$

$5000(1.0045^n - 1) = 650\,000(1.0045)^n \times 0.0045$

$5000(1.0045^n) - 5000 = 2925(1.0045)^n$

$5000(1.0045^n) - 2925(1.0045^n) = 5000$

$2075(1.0045^n) = 5000$

$1.0045^n = \dfrac{200}{83}$

$n \ln(1.0045) = \ln\left(\dfrac{200}{83}\right)$

$n = \dfrac{\ln\left(\frac{200}{83}\right)}{\ln(1.0045)}$

$= 195.878689$

$\therefore n = 196$ months #.

2

(iv) Total of loan over 360 months

= 360 × \$ 3649.95

= \$ 1 313 982

Total of loan if paying \$5000/month

= 196 × \$5000

= \$980 000

Interest saved = \$ 1 313 982 − \$980 000

= \$ 333 982 #

Question. 15. (15 marks)

(a)(i) 1

$$3x + 3x + 4y = 300$$
$$6x + 4y = 300$$
$$4y = 300 - 6x$$
$$y = \frac{300}{4} - \frac{6x}{4} \quad 1$$
$$\therefore y = 75 - \frac{3x}{2}$$

3 (ii)

$$A = 3x \times y$$
$$= 3x\left(75 - \frac{3x}{2}\right)$$
$$= 225x - \frac{9x^2}{2}$$
$$\frac{dA}{dx} = 225 - \frac{18x}{2}$$
$$= 225 - 9x \quad \text{— 1 for } \frac{dA}{dx}$$

Maximum area will occur when $\frac{dA}{dx} = 0$

ie $225 - 9x = 0$
$$9x = 225$$
$$x = 25 \quad \text{— 1 for } x$$

$$\frac{d^2A}{dx^2} = -9 < 0 \quad \text{for all } x$$

— 1 for test that it's a maximum.

$\therefore$ maximum area when $x = 25$ m.

Q15 Markers Comments

(a) this part was generally well attempted with most students scoring full marks.

(i) Common problems were:

- Some students didn't find the whole perimeter of the 3 children's blocks

Marking criteria:

- provides a correct solution — 1 mark.

(needed to see $4y = 300 - 6x$

$y = \frac{300}{4} - \frac{6x}{4}$ to get the 1 mark.

$y = 75 - \frac{3x}{2}$

(ii) Common problems were:

- students forgot the area of a rectangle formula

$A = l \times b$

$= 3x \times y$ of shaded area.

$= 3x\left(75 - \frac{3x}{2}\right)$

- Incorrect notation was evident across the year

Expectation was: $A =$, $A' =$, $A'' =$ OR $A =$, $\frac{dA}{dx} =$, $\frac{d^2A}{dx^2} =$

- Students forgot to show that $x = 25$ is a maximum from $\frac{dA}{dx}$ table or showing $\frac{d^2A}{dx^2} < 0$.

Marking criteria:

- correct solution — 3 marks
- $\frac{dA}{dx} = 225 - 9x$ — 1 mark (with working of course)
- $\frac{dA}{dx} = 0$, $\therefore x = 25$ — 1 mark
- test to show maximum area when $x = 25$ — 1 mark

1 (iii) when $x = 25$, $y = 75 - \frac{3 \times 25}{2}$

$\therefore y = 37.5$

$A = x \times y$

$= 25 \times 37.5$

$= 937.5 \text{ m}^3$ — 1 for correct area

(one of the children's blocks)

1 (iv) $1 \text{ ha} = 10\,000 \text{ m}^2$

$\therefore$ Amount left for Greg and his wife

$= 10\,000 - (3 \times 937.5)$

$= 10\,000 - 2812.5$

$= 7187.5 \text{ m}^2$ — 1 for correct area

(iii) Common problems were:

- students didn't read the question, area of one child's block was needed not the whole shaded area

Marking criteria:

- correct answer — 1 mark

(iv) Common problems were:

- students didn't know $1 \text{ ha} = 10\,000 \text{ m}^2$.

Marking criteria:

- correct answer — 1 mark

(b)(i) 2 $a = \ddot{x} = \dfrac{-14}{(t+4)^3}$

$t = 0$

$x = -0.75$ m

$v = \frac{7}{16}$ m/s

$= -14(t+4)^{-3}$

$v = \dot{x} = \int -14(t+4)^{-3} \, dt$

$= -14 \times \dfrac{(t+4)^{-2}}{-2} + c_1$

$v = \dfrac{7}{(t+4)^2} + c_1$

When $t = 0$, $v = \frac{7}{16}$: $\quad \dfrac{7}{16} = \dfrac{7}{(0+4)^2} + c_1$

$\dfrac{7}{16} = \dfrac{7}{16} + c_1$

$\therefore c_1 = 0$

$\therefore v = \dfrac{7}{(t+4)^2}$ # — 1 for velocity

$x = \int 7(t+4)^{-2} \, dt$

$= 7 \times \dfrac{(t+4)^{-1}}{-1} + c_2$

When $t = 0$, $x = -\frac{3}{4}$: $\quad -\dfrac{3}{4} = \dfrac{-7}{0+4} + c_2$

$c_2 = -\dfrac{3}{4} + \dfrac{7}{4}$

$\therefore c_2 = 1$

$\therefore x = \dfrac{-7}{t+4} + 1$ # — 1 for displacement

(b) a significant number of students found this part challenging

(i) Common problems were:

- poor integration skills
- many students rushing and getting wrong constants of integration
- $\frac{7}{4^2} = \frac{7}{4}$ a common error
- not simplifying $\frac{-14}{-2} = 7$ to make it easier to find c_1.
- those that got v wrong then struggled to integrate to find x.
- Some students integrated & differentiated at the same time (no marks awarded!)
- using $x = \frac{3}{4}$ instead of $x = -\frac{3}{4}$ to find c_2 (no marks awarded).

Marking criteria:

- 1 mark for correct velocity
- 1 mark for correct displacement

(2) (ii) passes through the origin when $x=0$

ie $\frac{-7}{t+4}+1=0$

$\frac{7}{t+4}=1$

$7=t+4$

$\therefore t=3$ seconds (1) correct for t

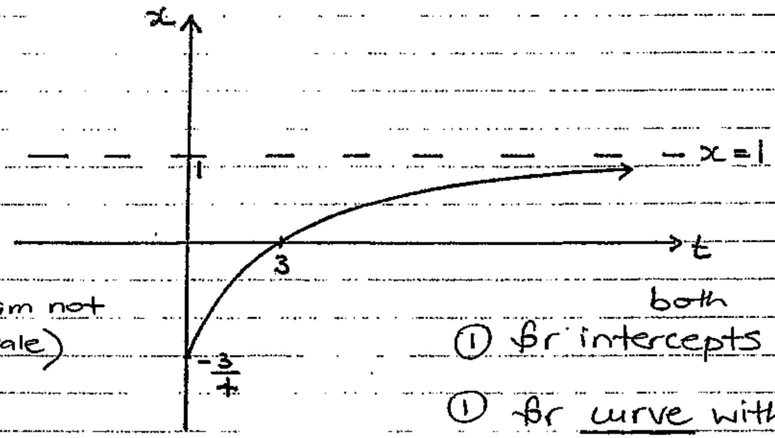
When $t=3$:

$V=\frac{7}{(3+4)^2}$

$=\frac{7}{49}$

$\therefore V=\frac{1}{7}$ m/s — (1) correct for V

(2) (iii) $x=\frac{-7}{t+4}+1$, where $t\geq 0$.



(diagram not to scale)

(1) for both intercepts

(1) for curve with asymptote at $x=1$ (curve had to have correct shape, not look like a straight line).

(ii) Common problems were:

- some students knew they needed to find t when $x=0$ and then substitute this t to find V.
- this was done successfully by some, but others found it harder as x was wrong from (i). They perservered and got the question correct for their wrong answer. Full marks were awarded as long as they showed working and I could check their solutions on the calculator.

CFPA = correct for previous answer.

Marking criteria:

- provides a correct solution — 2 marks
- $t=3$ seconds — 1 mark (or equivalent)
- $V=\frac{1}{7}$ m/s — 1 mark (or equivalent)

(iii) Common problems were:

- many students gave up with the graph
- students didn't recognise that $x=\frac{-7}{t+4}+1$ where $t\geq 0$ was an upside down hyperbola and shifted up 1.
- some drew the whole graph correctly with all branches and horizontal and vertical asymptotes and didn't realise domain was $t\geq 0$. [no marks were deducted!]
- graphs of straight lines were not awarded any marks
- only a handful across the form drew a perfect diagram as in solutions.

Marking criteria:

- provides a correct solution — 2 marks
- 1 mark for both intercepts
- 1 mark for curve with asymptote at $x=1$, curve had to have correct shape, not look like a straight line.

(c) $\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$

$$LHS = \frac{10^{3n} \times 25^{n+2}}{8^n}$$

$$= \frac{(2\times5)^{3n} \times (5^2)^{n+2}}{(2^3)^n}$$

$$= \frac{\cancel{2^{3n}} \times 5^{3n} \times 5^{2n+4}}{\cancel{2^{3n}}}$$

① for expanding the terms correctly

$$= 5^{5n+4}$$

ie $5^{5n+4} = 1$

$5^{5n+4} = 5^0$ — ①

equating powers with the same base:

$$5n+4 = 0$$
$$5n = -4$$
$$n = -\frac{4}{5}$$

— ① for correct answer

(c) a significant number of students found this part challenging

Common problems were:

- students forgot their indice work from year 11.
- students didn't use $1 = 5^0$
- students multiplied by 8^n and didn't see that they could cancel.
- most didn't see to rewrite 10^{3n}

$= (2\times5)^{3n}$

$= 2^{3n} \times 5^{3n}$ and so

the $8^n = 2^{3n}$ in the numerator and denominator cancel.

- question was easier if students equated the powers
- students made this question harder by using logs. (those that were competent got through to correct answer, others got completely lost.)

Marking criteria:

- provides a correct solution — 3 marks
- 1 mark for expanding the terms correctly
- 1 mark for $5^{5n+4} = 5^0$ (or equivalent)
- 1 mark for correct answer of $n = -\frac{4}{5}$

Question 10

a) i Given $V = Ae^{-kt}$

$30000 = Ae^{-5k}$ ①

$18000 = Ae^{-10k}$ ②

} 1 mark for either

① ÷ ② $\frac{30000}{18000} = e^{5k}$

$5k = \ln\frac{5}{3}$

$k = \frac{1}{5}\ln\frac{5}{3}$

1 mark

OR

$18000 = 30000e^{-5k}$

1 mark

$\frac{3}{5} = e^{-5k}$

$-5k = \ln\frac{3}{5}$

$k = -\frac{1}{5}\ln\frac{3}{5}$

1 mark

ii When $t = 5$, $V = 30000$

$30000 = Ae^{-5\times-\frac{1}{5}\ln\frac{3}{5}}$

$= A\times\frac{3}{5}$

$\therefore A = \$50\,000$

1 mark

iii $V < 1000$

$50000e^{-kt} < 1000$

1 mark

$e^{-kt} < \frac{1}{50}$

$\ln\frac{1}{50} > -kt$

$t > \frac{\ln 1/50}{-k}$

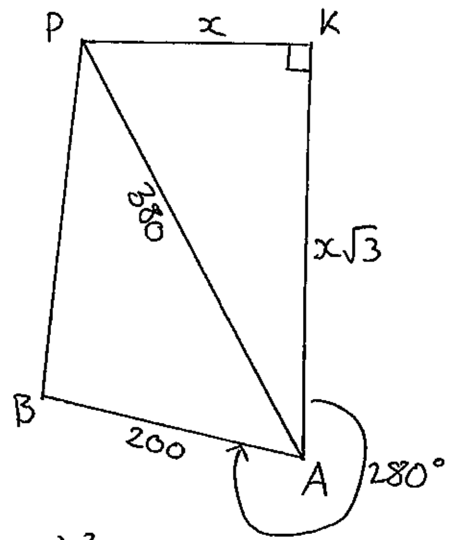
$> \frac{\ln 1/50}{-\frac{1}{5}\ln\frac{5}{3}}$

$> 38.29\ldots$

$\therefore$ It will take 39 whole years for the value to fall below \$1000

1 mark ($\frac{1}{2}$ mark for incorrect interpretation of answer)

b) i



There were many poor diagrams – this is crucial to the rest of this 5 mark question, so don't ru[sh]. If you get the diag[ram] wrong, you're going to have a bad time. Use pencil and a rul[er].

ii $x^2 + (x\sqrt{3})^2 = 380^2$

$x^2 + 3x^2 = 144\,400$

$x^2 = 36100$

$x = \pm 190$

but x is a length, so $x > 0$

$\therefore x = 190$

$\therefore AK = 190\sqrt{3}$ km

1 mark

iii $\tan\angle PAK = \frac{x}{x\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$\therefore \angle PAK = 30°$

$\angle PAB + 30° + 280° = 360°$ (angles at a point)

$\therefore \angle PAB = 50°$

1 mark

$PB^2 = 200^2 + 380^2 - 2\times200\times380\times\cos 50°$

1 mark

$= 86\,699.28$

$\therefore PB = 294$ km

iv $\frac{\sin\angle BPA}{200} = \frac{\sin 50}{PB}$

$\angle BPA = \sin^{-1}\left(\frac{200\sin 50}{PB}\right)$

$= 31.354\ldots$

$\doteqdot 31°$

$\angle KPA + 30 + 90 = 180°$ (angle sum of triangle)

$\angle KPA = 60°$

$$\ln(2x-5)=0$$
$$2x-5=e^0$$
$$2x=6$$
$$x=3 \qquad \therefore A(3,0)$$

question — you must <u>show</u> your working. Don't leave anything out!

when $x=6$
$$y=\ln(2(6)-5)$$
$$=\ln(12-5)$$
$$=\ln 7 \qquad \therefore B(6,\ln 7)$$

1 mark for showing each answer

ii $y=\ln(2x-5)$
$$2x-5=e^y$$
$$2x=e^y+5$$
$$x=\frac{e^y+5}{2}$$

1 mark

iii $A=\int_0^{\ln 7}\left[f(y)-g(y)\right]dy$
$$=\int_0^{\ln 7}\left(6-\frac{e^y+5}{2}\right)dy$$

1 mark

$$=\int_0^{\ln}\left(\frac{7}{2}-\frac{e^y}{2}\right)dy$$
$$=\frac{1}{2}\left[7y-e^y\right]_0^{\ln 7}$$

1 mark

$$=\frac{1}{2}\left[(7\ln 7-7)-(0-1)\right]$$
$$=\frac{1}{2}(7\ln 7-6)\text{ units}^2$$

or $A=A_1-A_2$
$$=6\times\ln 7-\int_0^{\ln 7}\frac{e^y+5}{2}\,dy$$

1 mark

$$=6\ln 7-\left[\left(\frac{7}{2}+\frac{5}{2}\ln 7\right)-\left(\frac{1}{2}\right)\right]$$
$$=6\ln 7-\frac{5}{2}\ln 7-3$$

1 mark

$$=\frac{7}{2}\ln 7-3$$
$$=\frac{1}{2}(7\ln 7-6)\text{ units}^2$$